Kuwait University | Math 101 | Date: November 24, 2008
Dept. of Math. &Comp. Sci. | First Exam (Make-up) | Duration: 90 minutes

**Calculators, mobile phones, pagers and all other mobile communication equipments are not allowed**

Answer the following questions:

1. Evaluate the following limits, if they exist:

(a) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^2+7}-\sqrt{3x+1}}{\sqrt[3]{x}-\sqrt{x}}$. (3 pts.)

(b) $\lim\limits_{x\to 0}\dfrac{\cos 7x-\cos 5x}{x^2}$. (3 pts.)

2. Find the $x$-coordinates of the points at which the function $f$ is discontinuous, where

$$f(x)=\begin{cases}\dfrac{x^2-1}{|x+1|} & \text{, if } x<-1,\\[2ex] \dfrac{\sin\left(\frac{\pi x}{2}\right)-1}{\sqrt[3]{x}} & \text{, if } -1<x<0,\\[2ex] \dfrac{1-x-(x-1)^3\sin\frac{\pi}{x-1}}{x(x-1)} & \text{, if } x>0.\end{cases}$$

Classify the types of discontinuity of $f$ as removable, jump, or infinite. (4 pts.)

3. State the domain of each of the following functions, then find the

(a) horizaontal asymptotes, if any, of $f(x)=\dfrac{\sqrt{7x^6+3x^4+2}}{x^3+1}$, (3 pts.)

(b) vertical asymptotes, if any, of $f(x)=\dfrac{2x^2-18x+36}{|2x+1|(3x-9)}$. (3 pts.)

4. Show that $f(x)=\dfrac{x^2+1}{\sin x+2}$, has a horizontal tangent line. (3 pts.)

5. Show that $f(x)=|\sin x|$, has a corner in $(-\pi,\pi)$. (3 pts.)

6. Use the definition of the derivative to find $f'(3)$, then find an equation of the normal line to the graph of

$$f(x)=\frac{x^2}{x+3}, \quad \text{at } x=3. \qquad \text{(3 pts.)}$$